A lot of practice making up stories, building representations of the stories and going through the steps will give the child a concrete understanding of what they are doing. Their understanding will be far deeper than simply knowing “the rules”.

Once a student understands the principles of two-place multiplication, he can easily proceed to problems which involve more places, i.e. 2345 x 4567. One of our favorite calculations is determining how many feet it is to the nearest star.

Many children, especially those with any spatial difficulties, get confused with two-place multiplication because they can not remember where all these numbers are supposed to be written. Numbers representing the units end up in the tens place and those which represent tens are often written in the hundreds place. We have tried graph paper to help the children place the numbers in the correct location, but the squares are too small. It is better to make your own graph paper with the table function of a word processing program. This way the size of the squares can be adjusted for the child.

Using the “Up the Hill” Manipulatives

One of the guiding principles we use at St. Michael's School is "from the concrete to the abstract". Abstract thinking is indeed important, but too often educators forget that the ability to think abstractly is founded on concrete experience. What is obvious to an adult is not necessarily obvious to a child. We have many years of life experience behind our ability to easily understand that 4 x 7 is the same as 7 x 4, or to determine whether to add, subtract, multiply, or divide in a particular story problem. The child has not yet had that experience.

The use of story and manipulatives gives the child a chance to experience, in a concrete way, the principles of mathematics. This concrete experience is what leads to understanding.

The use of games is also very productive. Every child is ready for a game. It is interesting to note that the Latin word *ludus*, means both school and game.

Worksheets have their place in re-enforcing the mechanics of arithmetic, but it is the concrete experience which produces a depth of understanding.

Using The "Up The Hill" Manipulatives

St. Michael's Orthodox School

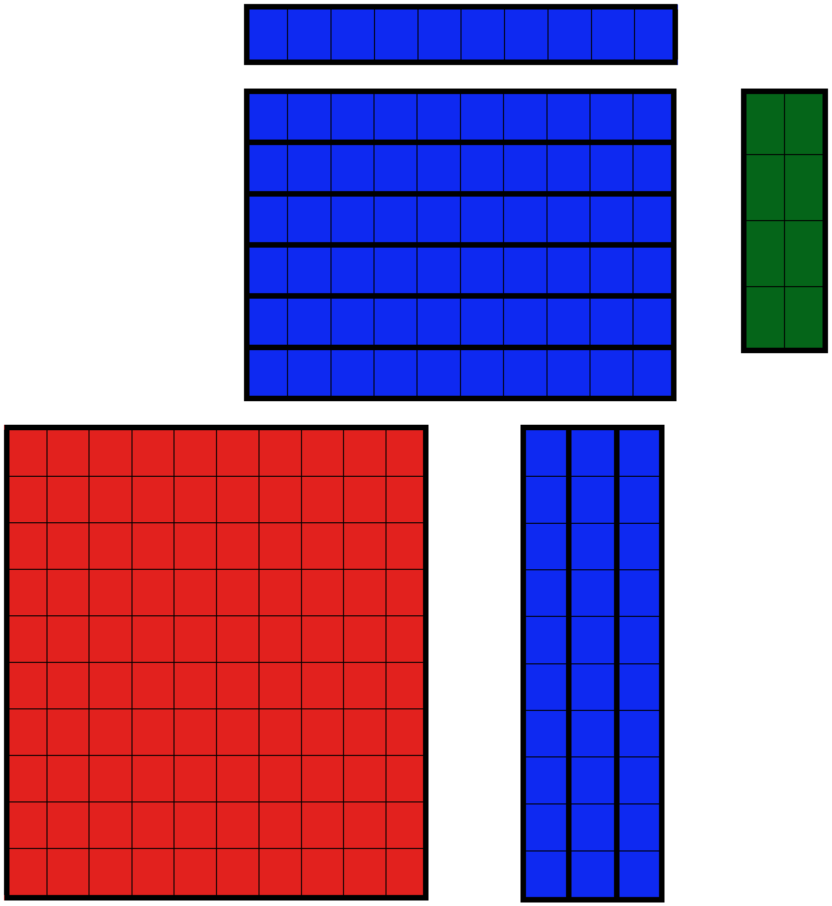
Since, there is no re-grouping necessary in the bottom level, the student simply records the 0 units, the 3 ten-bars and the 1 hundred flat is noted in the correct place. There are never any units beyond the upper level and this must be noted. **Not putting a 0 in the units place is a classic error that always results in an incorrect answer.**

The number in the bottom level (130) is the result of multiplying 13 x 10.

	1	3
x	1	6
	7	8
1	3	0

In this problem 78 and 130 are called "partial products". They must be added to obtain the total product. Since we have only 8 units, no re-grouping (carrying) is necessary. An 8 is simply written under the 0. When we add up all the ten-bars, we find we have 10 of them. These must be traded for one red hundred flat. After the ten-bars are traded, we have none left, so a 0 is placed under the 3. We do have two hundred-flats, so a 2 is placed in the hundreds column. The completed problem looks like this.

	1	3	
x	1	6	
1	7	8	Partial Product
1	3	0	Partial Product
2	0	8	Total Product



When the student is finished with any re-grouping in the upper level, he records what he sees on that level. In this case, there are 7 ten-bars and 8 units, 78. This is the result of multiplying 6 times 13.

	1	3	
x	1	6	
	7	8	←Upper Level
			← Bottom Level

USING THE "UP THE HILL" MATH MANIPULATIVES

These math manipulatives are named after one of the math games students love to play. The manipulatives can be used to practice the basic addition and subtraction facts, illustrate the concepts of "Borrowing" and "Carrying" (Re-grouping), and can provide a very enjoyable way of learning and practicing the multiplication tables. They are also used to illustrate the process of columnar multiplication.

I. Practice with basic addition facts and re-grouping with addition—Up the Hill, addition.

In this simple version of "Up the Hill", the players roll a pair of dice, add the value of each die and take that many of the small green cubes, the "units". If enough dice are available, the players can roll at the same time. The first player to reach a total of 100 wins the game. The final goal could be higher, depending upon time and situation.

Re-grouping practice is given because whenever a player has more 10 units, he must trade 10 of them in for a blue ten-bar. Whenever a player accumulates more than 10 blue ten-bars, he must trade 10 of them in for a red hundred-flat. Failure to trade in extra pieces results in losing the pieces that should have been traded.

Variations of this game include the use of more than two dice and using dice with numbers rather than dots.

II. Practice with basic subtraction and re-grouping with subtraction — "Down the Hill".

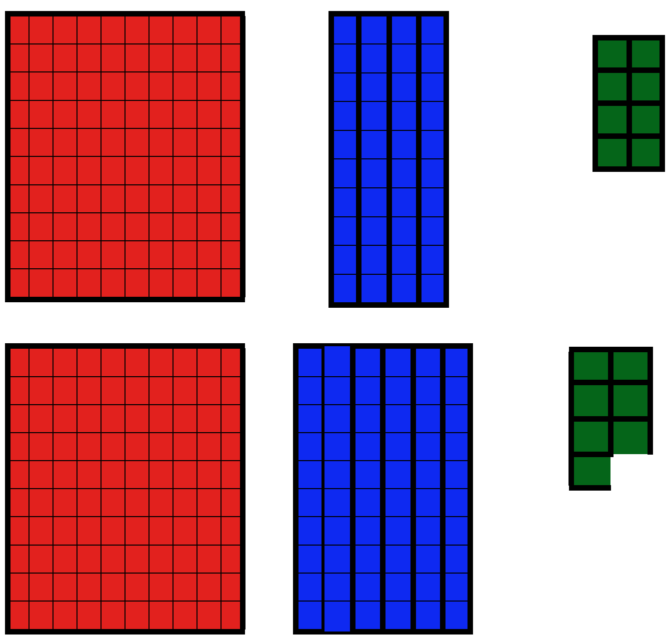
"Down the Hill" is essentially the reverse of "Up the Hill". The players start with the same amount of pieces, for example, a red hundred-flat. The goal is to be the first player to get rid of all the blocks. The players roll the dice, add the value of each die, and subtract the total from their pieces. On the first roll, each player will need to trade in the hundred-flat for 10 blue ten-bars. One of the ten-bars will then, most likely, need to be traded in for 10 green units. The players then remove from their pieces the total rolled on the dice.

This process continues until one of the players has gotten rid of all his pieces. The game can also be played with more than two dice, or with numbered dice.

III. Addition which involves re-grouping (carrying)

Use these manipulatives to demonstrate carrying in addition. For example, make up a story which would require adding 148 and 167. The problem would be written and shown with the manipulative like this.

$$\begin{array}{r} 148 \\ +\ 167 \\ \hline \end{array}$$



Ideally, the different color blocks would not overlap. They do here because of the width of the page.

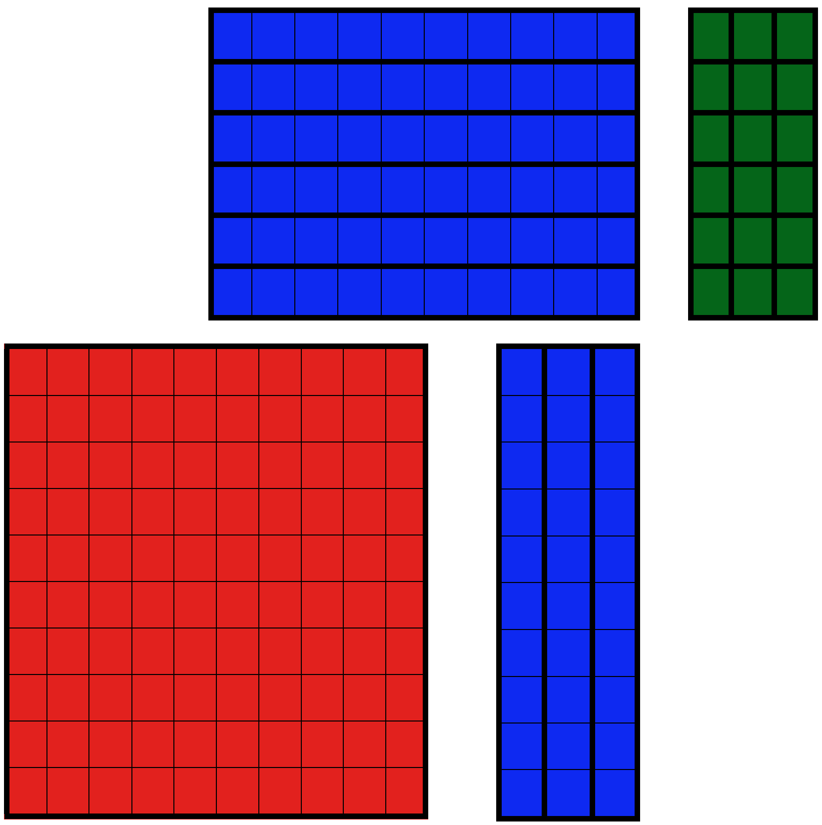
Notice that we now have four distinct rectangles of different sizes. The green rectangle shows what we get when we multiply 3 units by 6 units, or 18. The blue rectangle to the left of the green one is 1 ten by 6 units. This rectangle is, therefore, worth 60.

The blue rectangle on the lower level is 3 units over and 1 ten up for 30. Finally, the red rectangle is 1 ten over and 1 ten up for 100. These four rectangles are pictures of what results when we multiply 6 x3, then 6 x 1 (ten), 1 (ten) x 3, and finally 1 (ten) times 1 (ten).

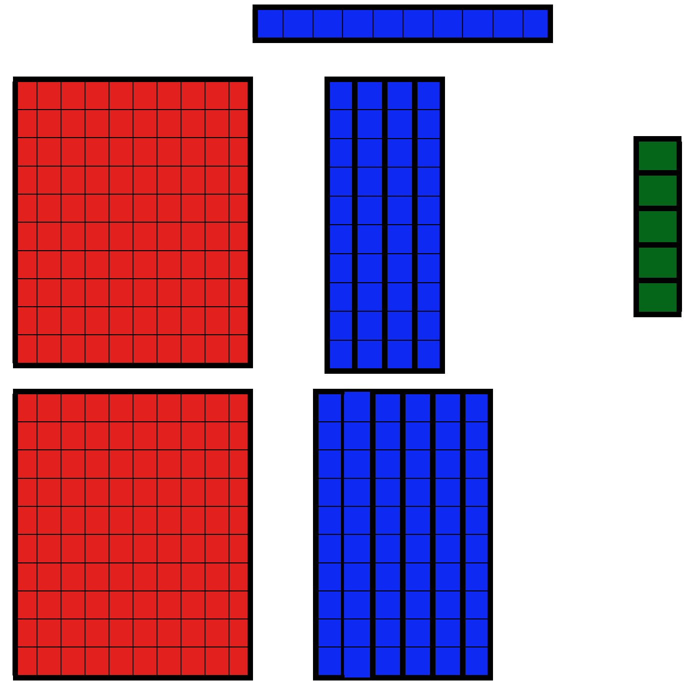
The next step involves making sure there are not more than nine blocks of one kind, starting with the units. Since there are eighteen units, ten of them must be traded for a blue ten-bar, resulting in the arrangement on the next page.

Students should be given plenty of practice building this type of representation with the manipulatives before taking the next steps. Simple story problems could be made up or taken from a math book. After building the rectangle (the representation will always be a rectangle), the student should always check to see that width is indeed the number of units of the top number and the height is the number of units of the bottom number. Starting with a representation that is not correct will cause confusion.

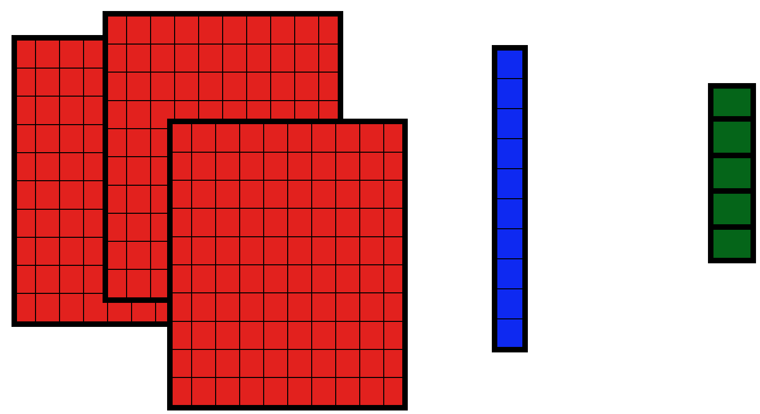
Once a student has learned to correctly build with the blocks, he can take the next step. Separate the blocks, as much as possible, into correct place value columns. In the example we are using, the ten bars and the units on top would be slid to the right until the 6 tens on the top are above the 3 tens on the bottom.



The units are pushed together and counted. Since there are 15 units, 10 of them will be traded for a blue ten-bar. This ten-bar is placed above the other ten-bars.



Next the ten-bars are counted. Since we have 11 of them, we can only keep 1 and trade in the other 10 for an hundred-flat. We see that we have 315.

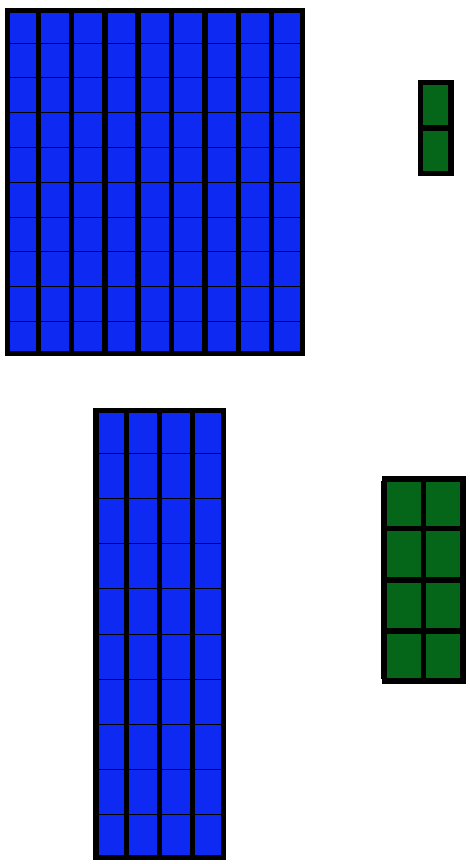


This type of physical manipulation of objects is very important for a child's <u>understanding</u> of arithmetic. It gives them an experience of the "why", which is far more valuable than simply "knowing the rules."

IV. Subtraction which involves re-grouping (borrowing)

Use these manipulatives to demonstrate borrowing in subtraction. For example, make up a story that would require us to subtract 48 from 92. The problem would be written as and shown with the manipulatives like this

$$\begin{array}{r} 92 \\ \underline{-\ 48} \end{array}$$



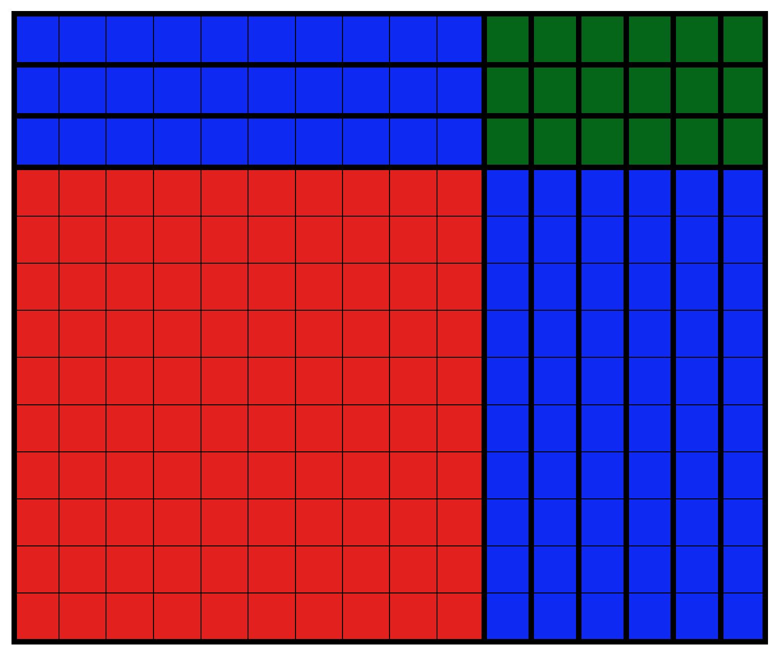
Each of the 13 vertical columns contains 16 units, and thus represents the apples picked by an individual child.

There are two important things to remember in building these representations. First, the top number (in this case 13), is always the number of units across. The bottom number (in this case 16), is always the numbers of units from the bottom of the representation to the top.

Secondly, build the representation from the bottom up and from the left to the right. In other words, any blue ten-bars in the rectangle would always be above or to the right of any red hundred-flats. The green units would always be in the upper right hand corner. For example, we could have written the problem

$$\begin{array}{r} 16 \\ \underline{\text{X}\ \ 13} \end{array}$$

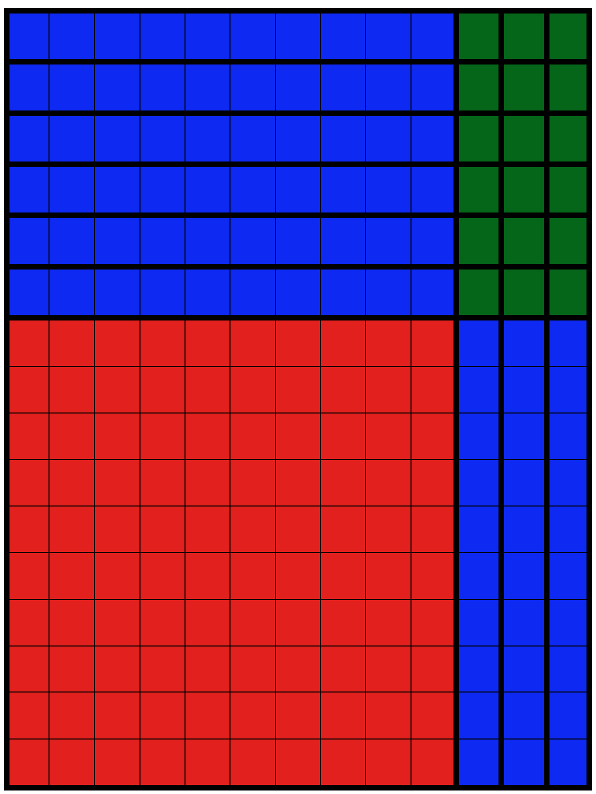
We would than have constructed a rectangle that looked like this.



top. A player must, however, flip either all the dice or none. Flipping an individual die is not allowed. The winner of the game is determined in the same way as regular Highest Product.

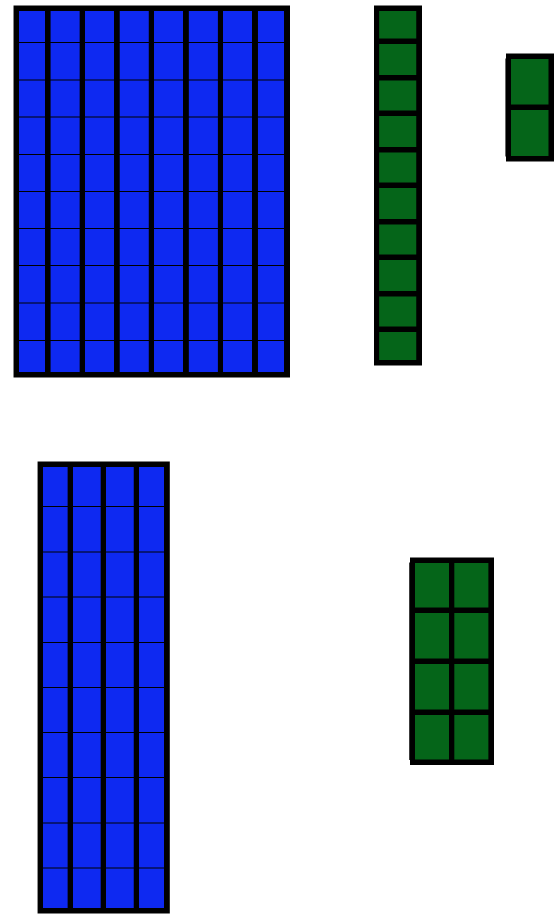
VI. Learning Two Place Multiplication

These manipulatives can also be used to demonstrate what is so often a mystery for children, two-place multiplication. Using the manipulatives shows the student, in a concrete, visual way, why each step is taken. The first thing a student does to make a picture of the problem with the blocks. For example, if 13 children each picked 16 apples from the trees in the orchard, how many apples were picked? The problem would be written and the manipulatives would be arranged like this.

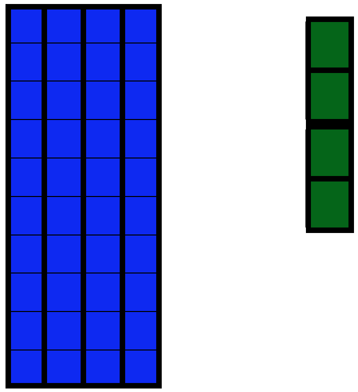


$$\begin{array}{r} 13 \\ \underline{\times\ 16} \end{array}$$

Since we can not take (subtract) 8 units from 2 units, we must trade in one of the ten-bars for 10 units. Now we have 8 ten-bars and 12 units.



We can show 8 units being subtracted by covering 8 units on the top with the 8 units on the bottom. This, of course, leaves 4 units uncovered. The same procedure can be used for the ten-bars. Four of the top ten-bars are covered with the 4 ten-bars on the bottom, leaving 4 ten-bars uncovered. The result is pictured on the next page.



After using the manipulatives with several simple problems, students can start writing the numbers which correspond to what has been constructed.

When assigning a sheet of addition or subtraction problems, we have found it very beneficial to the students to have them construct each problem first and then record in numbers what they are doing with the blocks.

V. Learning and Practice with the Multiplication Tables.

This version of "Up the Hill" is a very enjoyable way to learn and practice the times tables. It can be played by several players, each at his own level. Each player rolls a pair of dice and adds the value of each die. Each player then multiplies the sum by the times table he is learning. Players practicing the two times table multiply the sum by 2 and take that many blocks. Players practicing the five times table, multiply the sum by 5 and take that many blocks.

Each player has a "hill" or goal or a different height, depending upon the multiplication table being practiced. A child practicing the 2's has a goal of two hundred; a child practicing the 5's has a goal of five hundred. Therefore, all players have an equal chance of winning, despite the fact they may all be practicing different times tables.

The rule concerning trading in blocks when a player has more than ten of one kind applies in this game as well. For those students who are just beginning the times tables, we give them the Number Family Manipulatives to use as counters.

VI. Highest Product

This game is a variation of the regular "Up the Hill" which is played when the student know the times tables fairly well. In this version, all players use four dice, rather than two. Players roll the dice, then arrange them in pairs. The sums of each pair are then multiplied. The challenge is to arrange the four dice in such a way that the sum of the pairs yields the highest produce.

For example, if a players rolls 6, 4, 3 and 5, he has three possible combinations. Pairing the 6 with the 4, and the 3 with the 5 would give 10 x 8, or 80. Pairing the 6 with the 3, and the 5 with the 4 would give 9 x 9, or 81. Pairing the 6 with the 5, and the 3 with the 4 would give 11 x 7, or 77. The highest of the three products is 81.

If the same number appears on two dice, the player has only two possible combinations. A roll resulting in three of a kind, yields only one combination. Often there is not too much difference among the products, but sometimes the difference is significant.

The number of blocks required to win this version should be set before the beginning of the game. The game could also be played for a set amount of time, the winner being the player with the most blocks when the time is up.

VII. Highest Product— Flip the Dice

This a variation invented by our students. After rolling the dice, players have an option of flipping the dice and using the numbers on the bottom of the dice rather than those on the